Introduction: Big-O Notation

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Algorithmic Design and Techniques
Algorithms and Data Structures at edX

Learning Objectives

- Understand the meaning of Big-O notation.
- Describe some of the advantages and disadvantages of using Big-O notation.

Big-O Notation

Definition

$f(n) = O(g(n))$ (f is Big-O of g) or $f \preceq g$ if there exist constants N and c so that for all $n \geq N$, $f(n) \leq c \cdot g(n)$.

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f is bounded above by some constant multiple of g.

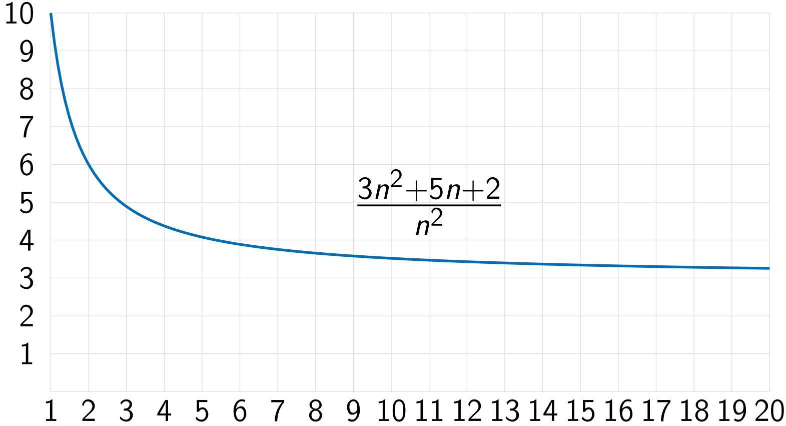
Big-O Notation

Example

$3n^2 + 5n + 2 = O(n^2)$ since if $n \geq 1$,
$3n^2 + 5n + 2 \leq 3n^2 + 5n^2 + 2n^2 = 10n^2$.

Growth Rate

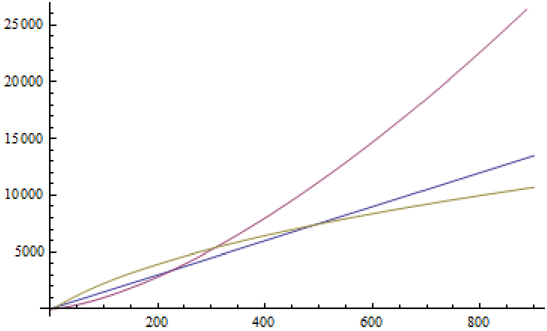
$3n^2 + 5n + 2$ has the same growth rate as n^2



Using Big-O

We will use Big-O notation to report algorithm runtimes. This has several advantages.

Clarifies Growth Rate



Cleans up Notation

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- $O(n)$ vs. $n + \log_2(n) + \sin(n)$.

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- $O(n\log(n))$ vs. $4n\log_2(n) + 7$.
  - Note: $\log_2(n), \log_3(n), \log_x(n)$ differ by constant multiples, don't need to specify which.

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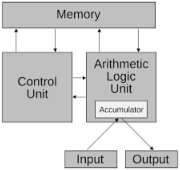
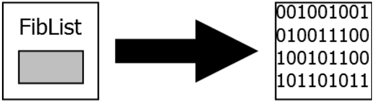
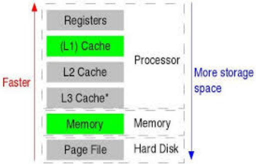
    - Note: $\log_2(n), \log_3(n), \log_x(n)$ differ by constant multiples, don't need to specify which.
- Makes algebra easier.

Can Ignore Complicated Details

No longer need to worry about:







Warning

- Using Big-O loses important information about constant multiples.
- Big-O is *only* asymptotic.