

Introduction: Using Big- O

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Algorithmic Design and Techniques
Algorithms and Data Structures at edX

Learning Objectives

- Manipulate expressions involving Big- O and other asymptotic notation.
- Compute algorithm runtimes in terms of Big- O .

Big- O Notation

Definition

$f(n) = O(g(n))$ (f is Big- O of g) or $f \preceq g$
if there exist constants N and c so that for
all $n \geq N$, $f(n) \leq c \cdot g(n)$.

Common Rules

Multiplicative constants can be omitted:

$$7n^3 = O(n^3), \frac{n^2}{3} = O(n^2)$$

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Smaller terms can be omitted :

$$n^2 + n = O(n^2), 2^n + n^9 = O(2^n)$$

Recall Algorithm

Function $\text{FibList}(n)$

create an array $F[0 \dots n]$

$F[0] \leftarrow 0$

$F[1] \leftarrow 1$

for i from 2 to n :

$F[i] \leftarrow F[i - 1] + F[i - 2]$

return $F[n]$

Big-*O* in Practice

Operation

Runtime

Big- O in Practice

Operation

create an array $F[0 \dots n]$

Runtime

$O(n)$

Big- O in Practice

Operation	Runtime
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for i from 2 to n :	Loop $O(n)$ times
$F[i] \leftarrow F[i - 1] + F[i - 2]$	$O(n)$
return $F[n]$	$O(1)$
Total:	

$$O(n) + O(1) + O(1) + O(n) \cdot O(n) + O(1) = O(n^2).$$

Other Notation

Definition

For functions $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$ we say that:

- $f(n) = \Omega(g(n))$ or $f \succeq g$ if for some c , $f(n) \geq c \cdot g(n)$ (f grows no slower than g).
- $f(n) = \Theta(g(n))$ or $f \asymp g$ if $f = O(g)$ and $f = \Omega(g)$ (f grows at the same rate as g).

Other Notation

Definition

For functions $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$ we say that:

- $f(n) = o(g(n))$ or $f \prec g$ if $f(n)/g(n) \rightarrow 0$ as $n \rightarrow \infty$ (f grows slower than g).

Asymptotic Notation

- Lets us ignore messy details in analysis.
- Produces clean answers.
- Throws away a lot of practically useful information.